Problem Set 8

Due Friday, November 9 at 10am

Submit Parts A-B on paper in Class or to Moodle. Submit Part C via Moodle only.

Part A - Conceptual

1. Describe in your own words why mass is the single most important variable in determining the properties of a star. Why does this single property dictate so many of the other properties?
2. In your own words, describe how, why, and in what proportion relative to other key properties the following stellar properties vary across the main sequence (e.g. which way do they increase and why? Do they scale linearly with mass? What processes dictate their variation?) . Connect your answers to specific equations or scaling relations wherever possible.
   a) Luminosity
   b) Surface/effective temp
   c) Wavelength of peak emission
   d) Radius
   e) Density
   f) Core Temperature
   g) Dominant source of opacity
   h) Dominant source of pressure support
   i) Lifetime on the main sequence

Part B - Quantitative

1. Apply the principle of mass-energy equivalence to the ionization and recombination of an electron in a hydrogen atom. Given the known binding energy required to ionize hydrogen from the ground state (n=1), what is the corresponding mass difference in gm between the bound (p+e) and unbound (p, e separated) state? In physics this is often called a "mass deficit or mass defect". Express the mass deficit as a fraction of the mass of a proton.
2. *Fusion of H to He*. The sun is powered by releasing binding energy in the fusion of hydrogen to helium. Whether via the proton-proton or CNO sequence the basic reaction is 4^1H morphs in ^4He, releasing nuclear binding energy (and losing mass) in the process.
   a) The mass of one proton ^1H is 1.6726 x 10^{-24} gm and the mass of one helium nucleus ^4He is 6.6447 x 10^{-24} gm. What is the mass difference in gm between 4 protons and one helium nucleus?
   b) Calculate the *ratio of the mass difference* to the original mass of the 4 protons. Express this as a percentage. What is the implication of this?
   c) Calculate the *binding energy* of a single fusion reaction of 4^1H to 1^4He, in both erg and eV.
   d) Use the above binding energy of one fusion reaction coupled with the solar luminosity to calculate *how many fusion reactions per second* occur in the solar core.

e) What is the corresponding total amount of *mass that is "disappearing" every second* in the sun's core? Give that mass both in gm and as a number/second of some common object, such as a dog, a tree, or a mountain.

3. Duration of current solar luminosity $\boldsymbol{\tau_{sun}}$:

a) Calculate the time in years that the sun could endure *if the entire body of the sun* eventually went through the fusion of hydrogen into helium.

b) What is the time *if only 10% of the sun's mass* eventually undergoes this fusion reaction during the sun's main sequence phase?

c) What is the corresponding *total amount of mass that the sun loses* in this process over the full course of its main sequence lifetime? Express this as a fraction of the current mass of the sun.

Part C - Computational

Work through the functions tutorial on the website. You are not required to submit it, however it is to your advantage to review it thoroughly and complete the exercises since it will help you to write the function outlined below, which you will need to do the lab on Friday.

Write your own Python function that takes as input the mass of a main sequence star and returns the rate of fusion reactions per second. As a test of whether it is working, show that you can approximately reproduce your answer to Part B, Problem 2b using your function. Submit your function definition and testing code as a single jupyter notebook.

Hints:

1) *You will need to translate the input mass to a luminosity in order to calculate the rate of fusion reactions.*
2) *Assume that all stars follow a scaling law that is intermediate to the low and intermediate mass star scaling laws that we derived in class, namely that* $L \sim M^4$
3) *Use ratios relative to solar quantities to turn the scaling relations into equalities.*