## Kepler's Laws and the Galilean Moons Lab

Work in groups of 2-3 and hand in one copy of the lab per group. All group members are expected to contribute equally. The prelab should be completed as homework BEFORE the lab period.

Objective: Familiarize yourself with observations of orbital motion. Discover and apply Kepler's Laws and the Law of Gravity.

Materials: Rulers (1 per group member), colored pencils (5 colors), graph paper, data handout

## Introduction

One of the great achievements of the astronomer Galileo Galilei was his identification of the four largest moons of Jupiter, now (appropriately) called the "Galilean moons" lo, Europa, Callisto and Ganymede. His observations were done with a small telescope and a great deal of patience. The observations that Galileo made of Jupiter, together with similar observations of Jupiter and the other planets in our solar system by Tycho Brahe, gave Brahe's apprentice Johannes Kepler the tools that he needed to develop the mathematical laws that we know today as Kepler's laws. You will find in this lab that you can use observations of Jupiter's moons to verify the basic principles described in Kepler's laws.

## Procedure

You should have received a packet with 25 images of Jupiter along with this lab. Each shows what you would see through a small telescope (today a "small telescope" still means bigger and of better optical quality than the one that Galileo himself used, which was essentially a sailor's spyglass!) if you were to have observed Jupiter in the night sky on the date and time indicated in the upper right of each image and from a special location on Earth, this is what you would have seen. The images have been color reversed, so bright things are dark and dark things are bright.

## Prelab - To Be Completed by EACH Group Member at Home BEFORE the Lab Period

This prelab portion involves a number or measurements to be made with a ruler. The more carefully you make these measurements, the easier your in-class work will be.

## 1. Label the Moons

If you examine them carefully, you will see that there are generally five objects in addition to Jupiter (the large, bright, banded object at the center) visible near it. Your first task is to go through each of the images and label these objects 1-5 in each image. Note that all five will not always appear in every image and that some of them will trade places over the course of the 50 hours of data shown, so look carefully. If you called one object "object 2 " in the first frame, you need to call it "object 2" throughout even when it switches places with other objects.

As you go through the images, you should also place a dot at the center of Jupiter in each image using a ruler.

## 2. Measure

Now, go through and measure the distance to each moon in millimeters from the center of Jupiter that you marked in the last step. Record each value in the table at the end of this lab. In cases where one (or more) of the five objects disappears, note that in the table by writing "gone" in the relevant box. There will be a couple of cases in which Jupiter itself is not visible in the image, in which case you should write "Jupiter gone" across the entire row of boxes. Mark all distances where the object is to the right of Jupiter as positive and all where the object is to the left of Jupiter as negative.

## 3. Predict

On the back of your table, explain the pattern of motion that you would expect a moon in orbit around Jupiter to make from our perspective here on earth - a "side view" of Jupiter. What would it look like if you were to graph the distance of each moon from Jupiter over time? It may help here to first consider what the orbits look like from a "top down" view, as though you were looking down on the solar system. Diagrams will also help you to visualize this. Use Kepler's laws to make an argument about how you might tell the difference between a moon that is close to Jupiter and one that is far away. Your prediction should be in the form of a full paragraph of complete sentences for full credit.

Once you have completed the three steps above, you are finished with the prelab portion of the assignment. Remember to bring your completed table to class! If you show up without it completed, you will not be placed in a group with others who have, so don't count on being able to "catch up" in class.

## Lab Portion - To Be Completed IN CLASS

## 1. Collaborate

Each of your group members should begin the lab with a completed table of measurements and a prediction of what pattern moons in orbit around Jupiter should make. Because you each completed this portion individually, the naming of your objects (1-5) may not be consistent originally. Begin by agreeing on an object naming scheme (which object is \#1, etc.). Each group member should fix their table according to this new scheme by changing the object numbers in the first row of the table where necessary. You should also discuss and debate your predictions before proceeding. All individual group members should write their names on their sheet of measurements and hand it in with the rest of the lab.

## 2. Average

One basic principle of science is that independent measurements that are averaged together are generally more reliable than single measurements. Average the measurements of your group members together for each moon at each time step and fill in the "group table" with these averages. This is where it is important that you have relabeled your moons to the
same numbering scheme as your other group members. If you were all careful, your measurements should be similar. If they are significantly different between group members, you should check them before proceeding. Perhaps the numbering scheme is off for one group member, or perhaps you don't agree which object is which when they switch places. Discuss all discrepancies and agree as a group before proceeding.

## 3. Plot

Compile all of the data from Step 2 and use it to make a chart (one per group, but all group members need to assist and should not just be watching one person do all of the work!) of the location of each moon relative to Jupiter. Your graph should follow the following guidelines:

1. The $x$ axis should show the distance from Jupiter in mm , and should include positive (to the right of Jupiter) and negative (to the left of Jupiter) values.
2. The $y$ axis should be placed at the location of Jupiter (an $x$ axis value of zero) and should show the time (from hour 1 to hour 50).
3. You should mark the location of each of the five objects at each time step (skip instances where it is "gone") with a small colored dot. Use five different colors for the five different objects and make a legend in one corner of your graph showing which is which.
4. Connect the path of each moon with a dashed line in that same color.
5. Follow the general guidelines for good graphs: use your space wisely (your chart should take up the whole page!), include a title and axis labels, etc.

## 4. Questions

Use the chart and the table that you made (as well as your instructor and TAs when you're stuck) to answer the following questions. Answer them on a separate sheet of paper. When you are finished, staple your individual and group tables, chart, and answers to the questions and hand that in to your instructor with all group members' names on it. For each of the following questions, use specific data from your chart and/or graph to support your answer. If you don't you WILL LOSE POINTS for completeness.

1. One of these things is not like the others! You may have noticed that one of the five objects near Jupiter over this 50 hour time span follows a very different pattern of motion from the other four.
a. Which object is it and how is it's motion different from the other four?
b. Is this object in orbit around Jupiter? How can you tell?
c. What might this object be? Make a guess and then design an experiment to test your hypothesis and describe it (hint: different types of objects in the sky move at different rates relative to one another. When would you come back to observe Jupiter again to verify or disprove your hypothesis?)

For each of the following questions, use specific data from your chart and/or graph to support your answer. If you don't you WILL LOSE POINTS for completeness.
2. The other four objects are the four Galilean moons of Jupiter.
a. Assign each of the objects that you've decided are moons a "Moon number" and record it as follows (on your separate sheet of paper)

$$
\begin{aligned}
& \text { Moon } 1=\text { Object } \\
& \text { Moon } 2=\text { Object ___ } \\
& \text { Moon } 3=\text { Object } \\
& \text { Moon } 4=\text { Object ___ }
\end{aligned}
$$

b. Do any of the four moons complete one full orbit of Jupiter over this time interval? If so, which? How can you tell?
c. Make a small graph (on your sheet of answers and not on graph paper is ok) showing one full orbit of a moon around Jupiter (a generic moon, not one of the ones you've observed, so you don't need specific numbers on the x or y axes).
d. On the same graph, sketch the path of a moon that is closer to Jupiter, and one that is farther from Jupiter and label all three.
3. The four moons of Jupiter show (or would show if we were to watch them for longer) the same general pattern of motion relative to Jupiter, but of course the four moons are also at different distances from Jupiter and therefore, according to Kepler's laws, orbit differently. One useful tool for judging their motions will be the speeds they are moving from your point of view, which are represented on the graph by the slope of the lines you plotted.
a. Do the moons move by the same distance each hour? What is the range of speeds ( $\mathrm{mm} / \mathrm{hr}$ ) for each of the moons?
Ex: Moon 1: 1 mm .hr at slowest and $5 \mathrm{~mm} / \mathrm{hr}$ at fastest
b. For the moon(s) that complete a full orbit, where are they relative to Jupiter in the diagram when are they moving the fastest and where are they when are they moving the slowest?
c. The orbits of the moons of Jupiter around Jupiter, like the orbits of the planets in the solar system around the sun, are very nearly circular, so this large change in apparent speed is NOT an effect of Kepler's laws/elliptical orbits. What is the real reason why they appear to change speed? Hint: consider the geometry of the system - how would your view change if you were looking from the "top down" (above the pole of Jupiter) rather than from the Earth looking toward Jupiter?
d. Approximately what percentage of a complete orbit have you observed for each moon? Justify your answer and explain how it is related to the range of speeds of each moon that you recorded in a.
4. The four moons that Galileo discovered in 1610 are named lo, Europa, Ganymede,
and Callisto. This table below shows the period and orbital radius for each moon. The period is the time for one complete revolution around Jupiter.

| Moon | Period (days) | Orbital Radius (km) |
| :--- | :--- | :--- |
| lo | 1.8 | 421,600 |
| Europa | 3.5 | 670,900 |
| Ganymede | 7.2 | $1,070,000$ |
| Callisto | 16.7 | $1,883,000$ |

For each of the following questions, use specific data from your chart and/or graph to support your answer. If you don't you WILL LOSE POINTS for completeness.
a. Which of your moons 1-4 is which? Justify your answer for each.
b. How are the orbital radii of the moons related to how they moved over the course of your observation? Use Kepler's laws in your explanation.
6. By analyzing images of Jupiter and its moons, you can determine values for the variables $D$ and $T$ in the equation below and solve for the mass of Jupiter, $M_{J}$.

$$
\mathrm{M}_{\mathrm{J}}=\frac{4 \pi^{2} D^{3}}{G T^{2}}
$$

In this equation, $D$ is the radius of the orbit of one of Jupiter's moons and $T$ is the time it takes the moon to complete one orbit (the orbital period). G, the constant of universal gravitation, and has a currently accepted value of: $G=8.65 \times 10^{-13} \mathrm{~km}^{3} / \mathrm{kg}-\mathrm{hr}^{2}$. Note that this equation looks a lot like Kepler's Third Law, but is modified to incorporate Newton's universal gravitation constant, which makes it apply to any central body that is being orbited by a much less massive object (not just the sun); e.g., The Hubble Space Telescope orbiting the Earth, a moon around a planet, one of the planets around the sun, or the sun around the center of our Milky Way galaxy. In all these cases the mass of the orbiting body is insignificant compared to the mass of the central body, and as you can see, its mass is not even included in the equation. If the mass of the orbiting body were significant, they would be orbiting around a common center, and a different equation would be needed.
a. As a practice problem, use the equation above to find the mass of the Earth given the following observational data: The period of the Moon around the Earth is 655 hr and the mean radius of its orbit is $384,000 \mathrm{~km}$. Once you have an answer, look up the real value or ask your instructor to verify that you are doing the calculation correctly. Make sure that you also attach a unit to the mass. Use the information you were given above to explain why that unit is valid.
b. Use your plot to estimate the "turn-around" point (the maximum distance from Jupiter) for each of the moons, in mm, from Jupiter.
c. In order to convert this "turnaround distance" to a real physical distance, you need a reference object. Luckily, Jupiter is the perfect candidate. It has a real diameter of $1.43 \times 10^{5} \mathrm{~km}$. Measure its diameter on your images and use it to estimate how much "real" distance 1 mm on your images represents.
Hint: This is a "conversion factor", which we talked about in class this week. As a reminder, when you have something of the form $A \mathrm{~mm}$ (in the image) $=B \mathrm{~km}$ (in real life), you can also say that $1 \mathrm{~mm}=B / A \mathrm{~km}$. In other words, $B / A \mathrm{~km}$ is the real distance represented by 1 mm in your images. Multiply all of your distances in mm by this conversion factor to get the real number of km between the objects

For each of the following questions, use specific data from your chart and/or graph to support your answer. If you don't you WILL LOSE POINTS for completeness.
d. Make a table listing each moon, your estimate for its turnaround distance in your images (in mm ) and it's real distance from Jupiter (in km) according to your conversion calculation (use the conversion factor you calculated in c). Save room for three additional columns in your table.
e. Again using your data only, estimate the period for each of the moons in hours. Your answer to 3d will help. Add this to the table you made in c (but don't forget to explain using data how you arrived at this conclusion).
f. You now have the period and radius for each of the moons. For each moon, plug these values as well as the constants into the equation for the mass of Jupiter and fill in that mass in the last column of your table.
g. Calculate the average of your four independent estimates for the mass of Jupiter.
h. The currently accepted value for the mass of Jupiter is $1.90 \times 10^{27} \mathrm{~kg}$. Determine the percent difference between this accepted value and your calculated (experimental) values using the following equation:

$$
\% \text { Difference }=\frac{\text { accepted value }- \text { calculated value }}{\text { accepted value }}
$$

Do it for each moon individually and for the average of all four.
i. Which of your estimates for the mass of Jupiter is the closet to the real vale? Why? Be specific, using the data from your table and from the table in \#4.
5. Wrap Up
a. What is the "big picture" of the chart that you made? Write a one sentence explanation describing what it shows.
b. The simulated observations here were taken over a 24 hr period. What location on earth does this represent? What is special about day/night in this place?
c. Galileo lived in Florence, Italy, where the sky moves nightly in the same way it does from amherst. Describe how they would have been different and what he would have had to do to overcome any obstacles created by his observing location.

## Individual Prelab

This portion must be completed by EACH group member at home BEFORE the lab period. If you come to class without this section completed, you will be asked to work on your own or with others who did not complete the prelab, and may not be able to finish the lab in the time allotted.

|  | Object 1 | Object 2 | Object 3 | Object 4 | Object 5 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Image 1 |  |  |  |  |  |
| Image 2 |  |  |  |  |  |
| Image 3 |  |  |  |  |  |
| Image 4 |  |  |  |  |  |
| Image 5 |  |  |  |  |  |
| Image 6 |  |  |  |  |  |
| Image 7 |  |  |  |  |  |
| Image 8 |  |  |  |  |  |
| Image 9 |  |  |  |  |  |
| Image 10 |  |  |  |  |  |
| Image 11 |  |  |  |  |  |
| Image 12 |  |  |  |  |  |
| Image 13 |  |  |  |  |  |
| Image 14 |  |  |  |  |  |
| Image 15 |  |  |  |  |  |
| Image 17 16 |  |  |  |  |  |


|  | Object 1 | Object 2 | Object 3 | Object 4 | Object 5 |
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| Image 19 |  |  |  |  |  |
| Image 20 |  |  |  |  |  |
| Image 21 |  |  |  |  |  |
| Image 22 |  |  |  |  |  |
| Image 23 |  |  |  |  |  |
| Image 24 |  |  |  |  |  |
| Image 25 |  |  |  |  |  |

Prediction:

## Group Table

This portion should be completed in class by each group. The measurements in this case should be the AVERAGE of the measurements made by individual group members of the positions of each object at each time step. Make sure to discuss and resolve discrepancies carefully before filling in the table.

|  | Object 1 | Object 2 | Object 3 | Object 4 | Object 5 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Image 1 |  |  |  |  |  |
| Image 2 |  |  |  |  |  |
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|  | Object 1 | Object 2 | Object 3 | Object 4 | Object 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Image 19 |  |  |  |  |  |
| Image 20 |  |  |  |  |  |
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| Image 25 |  |  |  |  |  |

