Math Skill #7: Fractions, Decimals and Percentages

One more important skill in improving your "number sense", or ability to reason with numbers and judge how reasonable/believable they are is to have a firm grip on percentages. Yes, this is a skill that you've seen before (many times!), but you may not have ever been trained regarding how to use them in everyday life. It's important to build up an intuition with percentages since they are very important to understand sales, risk statistics and many other things in real life, including how to calculate your grade on an assignment in Astronomy class!

We mentioned earlier that powers of ten were a good way to get a sense for how large or small a number is, which would allow you to compare it with other numbers. Well, one mathematical method for comparing two numbers, whether or not they're in scientific notation, is to divide them, effectively turning them into a fraction, decimal or percentage. In other words, fractions, decimals and percentages are, on a fundamental level, just a simple division, although of course you have to have a good understanding of what they mean in order to really reason with them.

Let's say these two numbers are A and B, so dividing them means A/B mathematically. The fractional representation of this is just the simplest that we can make A/B in whole numbers (for example 2/4 is more simply expressed as ½). When our numbers aren't nice whole numbers though, this won't do us much good (for example 63/71 doesn't have much meaning to us as a fraction) so we simply do the division in a calculator, giving us the decimal representation of the number. If we multiply this decimal by 100, we get a percentage. For example ½ is a fraction, 0.5 is a decimal and 50% is a percentage. Although these are all different, it is important to note that they are also the same - three different representations of the same mathematical idea.

Let's take a step back and think about what it means to compute a fraction, decimal or percentage. When you divide A by B, you are effectively asking: how many times does A fit into B? When A is larger than B, this answer will be larger than 1 and when A is smaller than B, this answer will be smaller than 1. In the case of percentages, we are usually (but not always!) in the latter situation, where A is the smaller number and B is the larger, so the number that comes out at the end is between 0 and 1, or 0 and 100%. This means that A fits into B *less than once*.

Since life is not always so simple, neither are fractions of percentages. Sometimes you might be asked to work backwards and figure out how much of something you need to make a certain percentage, etc. For example, if I gave you 30% odds that you are still reading this activity, how many people in my class of 35 students do I think have made it this far into my lengthy explanation of percentages? Well, 30% as a decimal is 30/100 or 0.3. If I multiply this by the total (35 students), I get $0.3\times35=10.5$ students.

Now, the .5 part is a little silly when we're talking about people since people only come in whole numbers, so maybe a better answer would be that I estimate that between 10 and 11 of my students are still reading the activity at this point.

The best skill that you can use when working with fractions, decimals and percentages is, as always, your common sense. Think about your answer before you put a box around it and move on. Does it make sense??

Sidebar: Percentages of Percentages
One particularly important skill with percentages is understanding how multiple percentages interact. This is often seen in real life consumer situations (for example, the sales racks at department stores often have 10% off something that's already 25% off, etc.) and in health care statistics. Understanding how percentages interact is particularly important for digesting information given to you about probability and/or risk.

For example, if I say that the risk of a woman developing breast cancer is elevated by 20% if they engage in some sort of bad habit that DOESN'T mean that 20 out of every 100 women who engage in that bad habit will develop breast cancer. It means that whatever the ORDINARY risk is for women (for example it's about 3.5% for women in their 60s), that risk is elevated by an additional 20% among women with this bad habit.

Does that mean that the risk for women with this bad habit is now 23.5% (which is what most people assume when they hear this)? NO! It means a 20% <u>elevated</u> risk, which means that you need to multiply the two percentages together. $0.2 \times 0.035 = 0.007$. Turn this into a percentage by multiplying by 100 and you get 0.7%.

Wait! That's lower than what we started with! Does that mean that the bad habit actually lowers a woman's risk of developing breast cancer? Again, NO! Because this is the <u>additional</u> effect of the bad habit, which adds to the previous 3.5% risk to give you a 3.5% + .7% = 4.2% risk for women in their 60s who have the bad habit.

1. At a casino, one of the games where the odds are most in your favor is roulette. A roulette wheel has 37 slots, numbered 0 to 35 plus 00 ("double zero"), which alternate between red and black (except for 00, which is colorless). If you bet on red or black, even or odd, the odds are almost 50/50 in your favor BUT the 00 is neither even nor odd, red nor black.
    a. Assuming that the roulette ball is equally likely to land in any one of the 37 spaces, what are the odds of winning when you bet on even or odd or red or black? Express your answer as a fraction, decimal and percentage.
    b. If the average bet at the wheel is $1 and the average number of bets placed per hour is 500 per wheel, how much profit might a casino

with three wheels expect to make over a 12 hour period? You may assume that everyone places one of the bets described in (a).

c. Explain in words why this is the MINIMUM profit a casino can expect to make over this interval.

------And now, some Astronomy questions ----------

2. The Earth gains about 10^8 kg from meteorite impacts every year. How much is this as a <u>percentage</u> of the mass of the Earth, which is 6 x 10^{24}kg?
3. The Earth loses ~1 x 10-7 "Earth masses" per year due to the solar wind's interaction with the upper atmosphere. Given this and your answer to question 1, is the Earth gaining or losing mass with time?
4. 620 million tons of Hydrogen are converted to Helium per second in sun (1 ton=2,000lb and 1kg=2.2lb). The reaction that produces one helium nucleus consumes four hydrogen nuclei, each of which has a mass of 1.67 x 10^{-27}kg.
   a. How many hydrogen atoms are consumed in the sun per second?
   b. How many helium atoms are created?
   c. What is 620 million tons as a percentage of the mass on the Sun if the Sun's total mass is 2x10^{30}kg?
   d. What is it as a percentage of the mass of the Earth, which has a total mass of 6x10^{24}kg?
5. In reality, only ~10% of the mass of the Sun's hydrogen will ever be consumed by fusion, because the Sun's core is the only place hot enough for fusion to occur.
   a. Assuming that the sun continues burning that fuel at the same rate for its whole lifetime, how long will the sun live in years?
   b. If the sun has been burning for approximately 4.6 billion years, approximately what percentage of the sun's total lifetime has already passed?

EXTRA CREDIT QUESTION

6. 3% of the solar flux comes in the form of neutrinos, each of which has an energy of ~2eV.
   a. How many neutrinos flow through the Earth per second?
   b. How many neutrinos flow through your body per second? (Hint: you will have to estimate the "cross-sectional area" of your body, or the percentage of a sphere centered on the sun that is taken up by your body – explain your reasoning in coming to this number in your answer)
   c. In your entire lifetime only about one neutrino will interact with the atoms in your body. Use this fact (and an estimate of the average human lifetime) to calculate the probability of a neutrino interacting with a particle of matter.