Math Skill #4: Manipulation of Numbers in Scientific Notation

There are some special rules to employ when doing addition, subtraction, multiplication and division in scientific notation, which are outlined below.

Multiplication and Division in Scientific Notation

The rules for multiplication and division of numbers in scientific notation are as follows:

$(A\times 10^{B})\times(C\times 10^{D})=(A\times C)\times 10^{B+D}$

$$\frac{A\times 10^{B}}{C\times 10^{D}} = (A\times 10^{B})\div(C\times 10^{D})=(A\div C)\times 10^{B-D}$$

Note that when you are multiplying, you add the exponents (the B in $A\times10^{B}$) together, and that when you are dividing you subtract the second (the one in the denominator, or bottom, of the fraction) from the first exponent. The coefficients (the A in $A\times10^{B}$) you can just multiply or divide normally.

*In other words, **remember to treat the coefficients and the exponents separately**. In many cases, you will need to use the tricks that you practiced last week to move the decimal after you do this in order to get the number back in proper scientific notation. THINK about it before you do this. Is your coefficient too small or too big? Do you need to increase it by adding one or more powers of ten, or decrease it by subtracting them? Do you have to make your coefficient bigger or smaller? Is your power of ten too small or too large?*

In fact, in general your best tool in evaluating answers to scientific notation problems is a little common sense. If you're dividing a very large number by a very small one, for example, do you expect your answer to be very large or very small? Generally speaking, ask yourself before you begin any problem whether you expect you answer to be large or small, positive or negative and compare the answer you get to your expectation.

Addition and Subtraction in Scientific Notation

Perhaps counterintuitively, addition and subtraction in scientific notation are much more difficult than multiplication and division. This is because in order to add the coefficients (the A in $A\times10^{B}$) of two numbers in scientific notation, the exponents (the B in $A\times10^{B}$) need to be the same. This means that you will have to use the rules that you learned in the Scientific Notation introduction last week in order to match the exponents before you add or subtract.

Example: $3.1\times10^{2} + 6.8\times10^{3} = ?$

The temptation for most people is to add the two coefficients (3.1+6.8=9.9) and to add the two exponents ($10^{2} + 10^{3} = 10^{5}$), and to say that this is equal to 9.9×10^{5} but note that this is NOT

true. These are small exponents, so you could verify this simply by writing out these numbers $3.1\times10^2 + 6.8\times10^3 = 310 + 6800 = 7110$. *This is DEFINITELY NOT the same thing as 9900000.*

Of course, when the exponents are more like 10^{24}, *you are not going to want to write out the numbers every time, so use your tricks for moving decimals AND YOUR COMMON SENSE to write the numbers with the same exponents. It doesn't matter which you choose. In this case, we could make both powers either* 10^2 *or* 10^3.

$$3.1\times10^2 + 6.8\times10^3 = 3.1\times10^2 + 68\times10^2 = (68 + 3.1)\times10^2 = 71.1\times10^2$$

OR

$$3.1\times10^2 + 6.8\times10^3 = .31\times10^3 + 6.8\times10^3 = (.31 + 6.8)\times10^3 = 7.11\times10^3$$

Note that once we made both exponents the same, now we can add the coefficients directly and just tack the power of ten on the end (the power of ten does not change!).

If we chose 10^2 *instead of* 10^3 *as the power of ten, there is one more step left in order to get our number in proper scientific notation. Again, using the tricks you practiced last week*

$$71.1\times10^2 = (7.11\times10)\times10^2 = 7.11\times10^3$$

Complete the following calculations. ***Show your work in the way laid out above****. Make sure your final answer is in proper scientific notation.*

$3.150 \times 10^{6} - 4.2 \times 10^{5} =$

$-1.592348 \times 10^{2} - 5.5 \times 10^{1} =$

$5.234 \times 10^{-2} + 8.1 \times 10^{-3} =$

$9.65 \times 10^{5} + 2.333 \times 10^{-1} =$

$-1.32 \times 10^{12} \times 7.94 \times 10^{2} =$

$6.77 \times 10^{-2} \times 3.56789 \times 10^{8} =$

$\frac{8.52 \times 10^{5}}{9 \times 10^{-2}} =$

$\frac{1.49598 \times 10^{11}}{2.998 \times 10^{8}} =$

Note: once you have completed the calculations above by hand (at least the exponent part, although you may wish to do the addition, subtraction, multiplication and division of coefficients with a calculator), you may wish to go back and check your final answers. If you do this BE CAREFUL not to be fooled by your calculator. If you type $3.1 \times 10^{2} \div 6.8 \times 10^{3}$*, without any parenthesis, your calculator will assume you mean* $(3.1 \times 10^{2} \div 6.8) \times 10^{3}$ *instead of* $(3.1 \times 10^{2}) \div (6.8 \times 10^{3})$*. Always put numbers in scientific notation in parenthesis!*

Another tip: most calculators have a shortcut for "times ten to the", which is the E or EE button, so 3.1×10^{8} *is 3.1 EE 8.*