<u>Math Skill #2: Powers of Ten</u>

In science, powers of ten are an important tool for understanding the ***<u>scale</u>*** *of a number, or its rough size. They are particularly powerful when comparing two quantities, or trying to figure out how much bigger one thing is than another. This is important in astronomy, where numbers vary in magnitude from the size of atoms (about 10^{-10} meters) to the size of the observable universe (about 10^{26} meters). Comparing those powers of ten, for example, tells you that the atom is 10^{36}, or 1,000,000,000,000,000,000,000,000,000,000,000,000 times smaller than the universe as a whole. If you stare at that number, it's nearly impossible to grasp it's size relative to other numbers. You could count how many zeroes it has and figure that each extra zero meant an additional 10 times larger, or you could look at it as a power of ten and the work will have been done for you!*

This skill of comparing large numbers is also useful, however, in life. For example, you often hear politicians talking about state budgets, government bailouts and the federal deficit in terms of thousands, millions, billions, and trillions of dollars. Understanding what these words mean in terms of powers of ten is a useful tool for understanding how much money you're really talking about, rather than just thinking of them all as really big numbers, which is what we tend to do. It's important to realize that a trillion (10^{12} important for numbers like the federal deficit) is three powers of ten, or 10x10x10=1000 times bigger than a billion (10^9, important for numbers like the world population). A billion is in turn three powers of ten or 10x10x10=1000 times bigger than a million (10^6, important for numbers like the size of a lottery win).

A power of ten is really exactly what it sounds like – the number 10 raised to some power (or "exponent"). The goal of a power of ten just to tell you roughly how big something is relative to other numbers. You can think of it in terms of how many zeroes there would be if you were to round to the nearest multiple of ten and count them. This is painstaking work once the number becomes big or small enough (especially if someone hasn't inserted a comma after every three zeroes!), which is why scientists use powers of ten. It's much easier in most cases to simply glance at the exponent on the power of ten rather than counting zeroes.

Associating a power of ten with a number is like asking the question: ignoring everything else, is the number you're talking about closer to 10 (10^1) or 100 (10×10 or 10^2)? Is it closer to 10,000 (10×10×10×10 or 10^4) or 100,000 (10^5)? 10 billion (10^{10}) or 100 billion (10^{11})?

Here are several things to note about powers of ten:

(1) *We are only talking about so-called "round" powers of ten here, so there are no numbers besides ones or zeroes to worry about. This means that we are indeed rounding, or approximating, our answers in many cases, but that's ok in that we are using powers of ten as a tool to understand the* ***rough*** *size of a number*
(2) *The "power" in the power of ten, is equivalent to the number of zeroes when you write the number out completely, so 10^6 is 1 with 6 zeroes after it, or 1,000,000 (one million!).*
(3) *The "power" in the power of ten could also be thought of as the number of tens you need to multiply together to get the number. To use 10^6 again, this means that 1*

million is the same thing as 6 tens multiplied together, or $10\times10\times10\times10\times10\times10=1{,}000{,}000=10^6$

(4) *This can also extend to negative powers, where a negative power of ten represents dividing the number 1 by that many 10s, rather than multiplying. For example* 10^{-1} *is* $1\div10$ *and* 10^{-2} *is* $1\div10\div10$*, or* $1\div100$*.*

Exercise 1: Rounding

For each number listed below, ROUND it to the nearest power of 10 (as in 10^X), and then write it out completely with all its zeroes. Then, write out the word that we commonly use for this number. Two examples are given below for your reference.

9,988,012	≈	10,000,000	=	10^7	= "ten million"
0.0000000004567	≈	0.0000000001	=	10^{-10}	= "one ten-billionth"

Note that the highlighted symbol above is a "squiggly" equals sign which means "approximately equal to". Using this symbol instead of a regular equals sign means that you're rounding, or approximating, the answer. This distinction is very important!

1,234	≈		=		=
26.1	≈		=		=
0.783	≈		=		=
89.098732	≈		=		=
0.003456	≈		=		=
89483217	≈		=		=
670,000	≈		=		=
0.000002	≈		=		=

Exercise 2: Comparing Powers of Ten

By comparing the difference in the exponents of two powers of ten, you can easily tell how many times bigger one is than the other. This saves you the trouble of having to write both numbers out and count zeroes. For each of the pairs of powers of ten listed, write a statement similar to the examples below where the words/numbers that you should alter are underlined.

Example 1: 10^9 vs. 10^6
$10^{\underline{9}}$ (one billion) and $10^{\underline{6}}$ (one million) are different by three powers of 10, or $10^{\underline{3}}$.

$10^{\underline{3}}$ = 1,000 = one thousand, so the powers of ten are telling you that one billion is one thousand times bigger than one million.

To state this another way, one billion is the same as one thousand millions.

Example 2: 10^{-2} vs 10^2
$10^{\underline{-2}}$ (one hundredth) and $10^{\underline{2}}$ (one hundred) are different by four powers of 10, or $10^{\underline{4}}$.

$10^{\underline{4}}$ = 10,000 = ten thousand, so the powers of ten are telling you that one hundred is ten thousand times bigger than one hundredth.

To state this another way, one hundred is the same as ten thousand hundredths.

1. 10^{12} vs. 10^9

2. 10^{-1} vs 10^{-3}

3. 10^1 vs 10^5

Exercise 3: Astronomical/Real World Examples

1. Round the underlined numbers to the nearest power of ten and write them in scientific notation
    a. The world population of approximately **7 billion**
    b. The US deficit of approximately **15 trillion** dollars
    c. There are **ten thousand** galaxies in the Hubble Ultra Deep Field, which was a **one million** second exposure of a region of space that initially appeared dark.
    d. A dust mite is **300 millionths** of a meter in size, while the atoms that make it up are only **trillionths** of a meter across.

2. Rewrite the underlined powers of ten below out in words (for example, replace 10^6 with “million”)
    a. Approximately $\mathbf{10^1}$ Jupiters would fit across the diameter of the sun (a straight line from one side to the other), while it would take $\mathbf{10^2}$ earths to cover the same distance.
    b. An Astronomical Unit (AU), the distance between the Earth and the Sun, is about $\mathbf{10^8}$ miles, while a parsec, which is a unit of distance used to measure the space between stars and galaxies, is about $\mathbf{10^{13}}$ miles.
    c. About $\mathbf{10^5}$ students take introductory astronomy at the college level in the US each year.
    d. Modern humans arose on Earth about $\mathbf{10^5}$ years ago, while the Earth (and Sun) were born from a cloud of interstellar gas about $\mathbf{10^9}$ years ago. The universe as a whole is about $\mathbf{10^{10}}$ years old.

3. For 1(c), 1(d) and 2(a)-(d) above, write a statement in words comparing the sizes of the two (or more) numbers given in each sentence. How much bigger is one than the other?

1(c)

1(d)

2(a)

2(b)

2(c)

2(d)