## Homework #9 Solutions

1. At a casino, one of the games where the odds are most in your favor is roulette. A roulette wheel has 37 slots, numbered 0 to 35 plus 00 ("double zero"), which alternate between red and black (except for 00, which is colorless). If you bet on red or black, even or odd, the odds are almost 50/50 in your favor BUT the 00 is neither even nor odd, red nor black.
   a. Assuming that the roulette ball is equally likely to land in any one of the 37 spaces, what are the odds of winning when you bet on even or odd or red or black? Express your answer as a fraction, decimal and percentage.
   *If you take away 00, there are 36/2, or 18 of each type (even, odd, red, black). The odds are thus 18/37 = 0.486 = 48.6% in favor of winning one of these bets.*
   b. If the average bet at the wheel is $1 and the average number of bets placed per hour is 500 per wheel, how much profit might a casino with three wheels expect to make over a 12 hour period? You may assume that everyone places one of the bets described in (a).
   *First, let's figure out how much money is bet. Using the information above,*

   $$3 \text{ wheels} \times 12\text{hours} \times \frac{500 \text{ bets}}{\text{hr}/\text{wheel}} \times \frac{\$1}{\text{bet}} = \$18{,}000$$

   *Now think about the answer to the last question. The casino wins 51.4% of these bets, so the expected profit for them is $18,000 x 0.514, or about $9,250*
   c. Explain in words why this is the MINIMUM profit a casino can expect to make over this interval.
   *This is the game and bet for which the odds are MOST in your favor, so you must anticipate that all other games are even more profitable for the casino (and less profitable for you!)*

------And now, some Astronomy questions ----------

2. The Earth gains about $10^8$ kg from meteorite impacts every year. How much is this as a <u>percentage</u> of the mass of the Earth, which is 6 x $10^{24}$kg?
   *The question is really: 'what is $10^8$kg as a percentage of 6 x $10^{24}$kg. The units are the same, so you should feel safe simply dividing them.*

   $$\frac{10^8 kg}{6 \times 10^{24} kg} = 0.167 \times 10^{8-24} = 0.167 \times 10^{-16} = 1.67 \times 10^{-17}$$

   *This is the decimal, so multiply by 100 to get it as a percentage*

   $$1.67 \times 10^{-17} \times 100 = 1.67 \times 10^{-15} = 0.00000000000000167\%$$

3. The Earth loses ~1 x $10^{-7}$ “Earth masses” per year due to the solar wind’s interaction with the upper atmosphere. Given this and your answer to question 1, is the Earth gaining or losing mass with time?
*Saying that the earth loses ~1 x $10^{-7}$ “Earth masses” is essentially saying that the Earth loses ~1 x $10^{-7}$ x 100 = 0.00001% of it’s mass every year.*

*To answer whether the Earth is gaining or losing weight with time – simply compare apples to apples. As a percentage of its weight, which is greater – loss (#3) or gain (#2)? Both are small percentages, but the “gain” rate is proportionally much smaller, which means that the Earth is slowly losing mass with time.*

4. 620 million tons of Hydrogen are converted to Helium per second in sun (1 ton=2,000lb and 1kg=2.2lb). The reaction that produces one helium nucleus consumes four hydrogen nuclei, each of which has a mass of 1.67 x $10^{-27}$kg.
   a. How many hydrogen atoms are consumed in the sun per second?

*First of all, an atom is very small and the sun is very large, so you should expect that LOTS of hydrogen atoms are required to power the sun. In other words, you should expect to get a big big number. This step of thinking about what you should get before you do your calculation is KEY with problems like this.*
*So, you are given the amount of hydrogen consumed per second in the problem, so the trick here is just to convert that mass (given to you in millions of tons) first into kilograms so that it matches the mass you’re given for a hydrogen nucleus.*

$$\frac{620\text{ Mton}}{1}\times\frac{10^{6}\text{ton}}{1\text{ Mton}}\times\frac{2000\text{ lb}}{1\text{ ton}}\times\frac{1\text{kg}}{2.2\text{ lb}}=5.64\times10^{11}kg$$

*Now that you have the total mass being burned in kg and the mass of an individual hydrogen atom in kg, all that’s left is to divide!*

$$\frac{5.64\times10^{11}kg}{1.67\times10^{-27}kg}=3.38\times10^{38}$$

*First, note that as expected, we got a really really really big number. If, by mistake, you had done this fraction backwards, note that you would have had a really really really small number. But you wouldn’t have been fooled by that, because it doesn’t make any sense! Next, not that our units are the same in the top and bottom of this fraction, so our answer is unitless. But I said that I never want you to give me a number in this class without a unit so you need to step back for a second and THINK about what you just did. Note that you divided the TOTAL mass being burned by the mass of one individual hydrogen atom, which gives you the **<u>total number of hydrogen atoms</u>** being consumed, so the final answer is **$3.38\times10^{38}$ hydrogen atoms.***

   b. How many helium atoms are created?

*Consider the net reaction that is going on here.*

$$4H \rightarrow 1He$$

*This tells you that for every four Hydrogen atoms consumed in the sun, 1 Helium atom is created. And you already know the number of hydrogen atoms being consumed per second because you solved for it in (a). Simply multiply this by ¼ (or, alternatively, divide by 4) to get the number of Helium atoms created,* ***8.45x10$^{37}$ Helium atoms.***

*Note that again logic prevails – you already know that this number should be smaller than the number you got in (a), since it takes 4 hydrogen atoms to make 1 helium.*

c. What is 620 million tons as a percentage of the mass on the Sun if the Sun's total mass is 2x10$^{30}$kg?

*You know by now that percentage means that you need to take a ratio of the two numbers (IN THE SAME UNITS!) and then multiply by 100, but which goes on top and which on bottom? In this case, you could try them both and see which one comes out to a number between 0 and 100. You could also take a step back and think about what makes sense in terms of the numbers. You know that you'rep;; nn taking a small piece and calculating what it is as a percentage of a whole, so you should be dividing the smaller number by the larger. If you divide 620 million tons (but converted to kg, which you did in a) by the mass of the sun and then multiply by 100, you get* ***2.82x10$^{-17}$%.*** *This may make more sense to you if you write it out all the way. The mass of hydrogen being burned as a percentage of the whole mass of the sun is 0.00000000000000000282%*

d. What is it as a percentage of the mass of the Earth, which has a total mass of 6x10$^{24}$kg?

*This is the same procedure except that this time you should divide by the mass of the earth. You will get 9.39x10$^{-14}$ or, when you multiply by 100, 0.0000000000000939%*

5. In reality, only ~10% of the mass of the Sun's hydrogen will ever be consumed by fusion, because the Sun's core is the only place hot enough for fusion to occur.
   a. Assuming that the sun continues burning that fuel at the same rate for its whole lifetime, how long will the sun live in years?
   *If the sun has 2 x 10$^{30}$ kg of mass in it (all hydrogen, we're assuming), but only 10% of it is available to be burned, then a total mass of 2 x 10$^{30}$ x 0.1 = 2 x 1029kg is available as fuel for nuclear fusion. If we divide this total mass of fuel by the rate at which that fuel is burned, we get an estimate of the lifetime of the sun.*

$$\frac{2\times10^{29}kg}{5.64\times10^{11}kg/sec} = 3.55\times10^{17}sec$$

$$\frac{3.55\times10^{17}sec}{1}\times\frac{1\ hr}{3600\ sec}\times\frac{1\ day}{24\ hr}\times\frac{1\ yr}{365\ days} = 1.13\times10^{10}years$$

   *or 11 billion years! Whew! That seems like a long time!*

   b. If the sun has been burning for approximately 4.6 billion years, approximately what percentage of the sun's total lifetime has already passed?

*But how long is it really compared to how long the sun has already been living? At this point you already have both numbers – the current age and the total lifetime both in billions of years. Simply divide the two and multiply by 100 to get a ratio!*

$$\frac{4.6 \; billion \; years}{11.3 \; billion \; years} = .407 \times 100 = 40.7\%$$

*Less than halfway. Whew again!*