Homework #8 Solutions
Due in class Wednesday, March 28

Answer parts 1 and 2 on a separate sheet of paper. You do NOT need to print this to hand in with your answers.

Part 1. Definitions

1. For each of the terms below, write two sentences. The first sentence should be a definition **in your own words**, and the second should be an explanation of how this concept is significant in Astronomy.

a) Differentiation
Differentiation is the process in which denser materials sink towards the center of a molten body (like a forming planet). It is important because it explains why large objects in our solar system, such as the Earth, have light materials at their surfaces and denser materials at their core. In particular, it explains how Earth and the other planets in our solar system got their dense, molten metal cores, which create a planetary magnetic field when the planet is rotating.

b) Protoplanetary Disk
A Protoplanetary disk is the pancake-like disk of material surrounding a young star. It is a result of the collapse of a molecular cloud to form a star, and can be thought of as "leftover" material from star formation. It is composed of gas and dust (solids) and is important in astronomy because it is the material out of which planets are made!

c) "Dust"
In Astronomy, "dust" means any solid material (as opposed to gas or liquid). We find it all over in astronomy, including in the space between planets, stars and galaxies. It is important because it provides the solid material out of which terrestrial planets (and probably giant planet cores) form.

d) Infrared Excess
Infrared excess refers to the extra "bump" of infrared emission that is visible in the spectra of young stars in addition to the continuous (blackbody) spectrum of the star itself. It indicates cool(er than the star) material. It is important because we "spot" young stars by looking for infrared excesses.

2. Assuming that they start with the same protoplanetary disks, which of the following two systems would have LESS time in which to form planets? Justify your answer in words and using the equations for the momentum of photons and massive particles in your explanation.
(a) A system with a sun-like star
(b) A system with a hotter star that emits more high-energy light

As the problem states, the hotter star emits more high energy light. That light therefore has shorter wavelengths on average than the light coming from a sunlike star, since energy and wavelength are inversely proportional (according to the equation E=hc/λ). Since the momentum of a photon is ALSO inversely proportional to wavelength (according to p=h/ λ), the momentum of photons also increases as the wavelength decreases. A particle of gas or dust with a given mass, therefore, will have more momentum transferred to it when it absorbs this shorter wavelength photon. This results in a higher outward velocity according to the equation p=mv, where momentum and velocity are directly proportional. Thus, the material is lost more quickly from around the hotter star.

Part 2. Math Skill #6: Dimensional Analysis Problems

1. If carpet costs $5/square foot and you need to carpet a room that's 5ft x 10ft, how much will it cost you?
   (a) I was given a quantity with units of cost per area and two dimensions of length.
   (b) The solution should have units of cost
   (c) I need to get rid of the units of area by multiplying by an area so that area appears on the top and the bottom of the fraction, leaving me with just cost
   (d) I will need to turn my two length units into a unit of area by multiplying them together so that the units on both the top and bottom are square feet
   (e) $\frac{\$5}{ft^2} \times (5ft \times 10ft) = \frac{\$5}{ft^2} \times (50ft^2) = \$250$
   (f) You know that this makes sense because the area is greater than 1 square foot and the cost is greater than $5.
2. The current world population is 7 billion. If the population continues to grow at a rate of 200,000 people per day, then what will the population be in 50 years?
   (a) I was given a quantity with units of population, one with units of population per time and one with units of time.
   (b) The solution should have units of population (people)
   (c) I need to get rid of units of time by multiplying my population per time number by a time so that the units of time cancel and I am left with just population.
   (d) I will need to convert years to days or vice versa so that my time units match and cancel
   (e) $\frac{200{,}000\ people}{day} \times 50 years$ *the units won't cancel until we convert 50 years to days*

   $$\frac{50 years}{1} \times \frac{365\ days}{1\ year} = 18{,}250\ days$$

   Now we can put this in place of the 50 years in the original equation and out units will cancel

   $\frac{200{,}000\ people}{day} \times 18{,}250 days = 3.65 \times 10^5\ people$*, or 3.65 billion people*

   Note that you could have actually done this all in one step if you had taken an even more careful look at the units

   $$\frac{200{,}000\ people}{day} \times 50 years \times \frac{365\ days}{1\ year} = 3.65 \times 10^5\ people$$

(f) *Note that this alone does NOT make sense because it is LESS than the population you started with. This should be a clue that you need to do something else, which is to add this INCREASE in the population to the original value. The population in 50 years will be 10.7 billion people if this projection is correct.*

3. Bob has only $20 in his pocket and he is 150 miles from home. His car gets 35 miles per gallon. What is the most that gasoline could cost per gallon for Bob to be able to make it home with his $20?
   (a) *I was given a quantity with units of dollars, one with units of miles, and one with units of miles per gallon.*
   (b) *The solution should have units of cost per gallon.*
   (c) *I need to get rid of units of distance, and end up with dollars in the top and gallons in the bottom of the fraction, so I should divide the money he has by the miles to home to get his budget per mile. If I multiply this by miles per gallon then the miles will cancel and I'll have units of cost per gallon.*
   (d) $\frac{\$20}{150mi} \times \frac{35mi}{gal} = 54.67\ per\ gallon$
   (e) *I could verify that this makes sense by manipulating the numbers a little further. If the cost per gallon is $4.67, then $20 will get me 4.28 gallons of gas. At 35 mi/gal, this will get me 150 miles.*

4. It is approximately 3000miles from coast to coast in the US. If you embark on a cross country road trip where you bike 8hr/day at a speed of 20mi/hr, how long will it take you to cross the country?
   (a) *I was given a quantity with units of mi, one with units of hr/day and one with units of mi/hr.*
   (b) *The solution should have units of days*
   (c) *To end up with only days, I need miles and hours to cancel and days in the top of the fraction, so I'm going to divide the cross country distance by the speed to give me the time in hours and then divide that by the number of hours per day that I'm biking to get the total number of days.*
   (d) $3000mi \div \frac{20mi}{hr} = 150hrs$

   $150hr \div \frac{8hr}{day} = 19\ days$

   (e) *This makes sense because 20 mi/hr x 8 hr/day x 19 days is approximately equal to 3000 miles.*

5. It costs Suzie $3 per gallon to produce lemonade at her lemonade stand, and she sells 8oz cups for 25cents each. If she makes 3 gallons one Sunday and sells it all, how much profit will she make? (Note: there are 8oz in one cup and 16 cups in a gallon)

(a) I was given a quantity with units of dollars per gallon, one with units of ounces (volume), one with units of cents and another with units of volume.
(b) My answer should be in units of dollars or cents (money)
(c) - (e) Because there are lots of things with the same units in what I was given, I need to be extra cautious here.
Let's start by calculating Suzie's manufacturing cost for 3 gallons of lemonade.

$$3gal \times \frac{\$3}{gal} = \$9$$

Now let's find out how much money she made by selling the whole 3gal. To do this, we need to convert gal to oz.

$$3gal \times \frac{16c}{gal} \times \frac{8oz}{1c} = 384oz$$

Now if she sold this volume at 25c per every 8oz, then

$$384oz \times \frac{25cents}{8\ oz} = 1200\ cents$$

Notice that because the volume of the cups she was selling was in face 1 cup, you could have saved the trouble of converting to ounces if you thought through the problem extra carefully before beginning.
Convert this to dollars

$$1200cents \times \frac{\$1}{100\ cents} = 512$$

so Suzie made $12 – $9, or $3 of profit, which makes sense because everyone knows that lemonade standers aren't in it for the money.

Exercise 2: Use this same kind of reasoning for Astronomy problems.
Solve the following problems using whatever method you are most comfortable with.

1. It takes light 8 min to reach the Earth from the Sun, which is 1 AU away. How long does it take light to reach Uranus, which is 20AU from the Sun? Write one sentence explaining why your answer makes sense.
   Uranus is 20 times farther away, so it should take the light 20 times longer to get there. This mean it takes 8min x 20 = 160min for light from the sun to reach Uranus. Notice that that's almost 3 hours!

2. The wavelength and frequency of light are related via the speed of light. Write a formula for the speed of light that includes only wavelength and frequency and use units to justify your answer.
   Since speed needs to have units of distance over time, distance units need to be on top and time on the bottom. Since wavelength has units of meters and frequency has units of "per second" (Hertz), all we have to do it multiply them together to get meters per second – a speed! The formula is therefore $c=\lambda f$

3. Someone tells you they heard that the area of the Milky Way Galaxy is 100 thousand light years.
   a. Why do you know, based on units alone, that this can't be true?

An area is a distance squared, so it should have units of distance squared (like square meters or square lightyears)

b. Why is "Area" not a good measurement for a 3D object like the Milky Way anyway? Draw a picture to support your answer.
Area only measures the two dimensional coverage of an object, but a galaxy, like so many things, is three dimensional and therefore should be measured with a volume. Your picture should show that the galaxy has thickness, in addition to length and width.

c. For what kinds of objects is area a good indication of their size? In other words, what kinds of things should you quote areas for instead of lengths or volumes?
Areas should be used for two dimensional (flat) objects – like pieces of paper or carpet.

4. Someone else tells you that the volume of the observable universe is about 3 trillion trillion million cubic parsecs. What is this as a power of ten and why can you say that the parsec is definitely a unit of distance (even if you didn't remember that to begin with) based on this measurement of volume?
Trillion is 10^{12} and Million is 10^{6}, so a trillion trillion million is $10^{12}x10^{12}x10^{6}=10^{12+12+6}=10^{30}$! We know automatically that parsec must be a unit of distance, because volume has units of distance cubed and the volume they quote is in cubic parsecs.

5. The nearest star to our sun is about 3 light years away. If you broadcast a radio signal from Earth and an alien civilization intercepted it and sent another radio signal immediately back, how much older would you be when their reply arrived relative to how old you were when you sent the signal?

Since light travels at the speed of light, it takes it 3 years to travel 3 lightyears (although it's important to note here that a lightyear is a distance NOT a time). So it takes the signal 3 years to get there and 3 years to get back, for a total of 6 years. You'll be 6 years