Math Skill #9: Scale Factors

We've already discussed unit conversions in class, and in order to complete this lab you will need to use them. But in order to understand scale modeling, we also need to build in the concept of a **scale factor**.

A scale factor is a number that tells us how many times smaller a model is than the real thing. If you've ever looked closely at a map, you may be familiar with a scale factor. For example, 1:10,000 is a common scale factor for a hiking map and you would find it written at the bottom. In fact, every (decent) map should have a scale factor written on it somewhere in the form of 1:X, where X is some large number (generally a round power of 10 like 1,000 or 1,000,000, but sometimes not).

A scale factor of 1:10,000 means that the true terrain is 10,000 times larger than that represented on your map, or that if you were to blow up your map by a factor of 10,000, it would be the same size as the real terrain. Obviously this would be impractical and would defeat the purpose of having the portable map in the fist place, so the scale factor is important in telling you the true distance between points. If you don't know whether the scale factor of your map is 1:1,000 or 1:10,000, then you don't know whether it should take you 10 minutes or several hours to walk a distance on the map.

Scale factors are always **unitless** which makes sense if you think about it. It shouldn't matter if you measure distances on a map in inches (US) or in cm (Europe), the factor by which you have to multiply your measurement to get the true distance should be the same. If you measured in inches and multiply by your unitless scale factor, you'll get the true (probably very large) distance also in inches. This also goes for cm, m, ft or any other unit of length you could use to measure.

One thing that mapmakers often do is provide a key that shows the size of a real unit of distance, for example one mile or one kilometer, on the scale of their map like in Figure 1 below. To do this, they use unit conversions! They calculate how big their large unit (1 mile) if you were to measure it in a small unit, like centimeters or inches. They then divide this very large number (which, remember, still represents the TRUE distance of 1 mile, but in vary small units) by the scale factor to get the equivalent of that distance on their map.

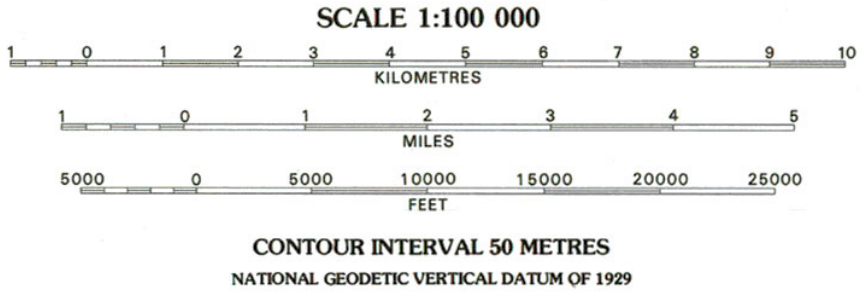


Figure 1: A sample scale from a real map showing the scale factor at the top and the equivalents of several types of real distances on the scale of the map.

Using the information above, your "Math Cheat Sheet", class notes, and what you learned from your Introduction to Unit Conversions exercise, answer the questions on the following pages. **Show your work and answer in complete sentences for full credit!**

1. Calculate how many inches are in one mile.

2. Your answer above is for a 1:1 scale map. If you were to create a map that was the same size as reality, each mile would represent this number of inches (the true number of inches in one mile). If you were to represent this distance (1 mile) on a map that had a scale factor of 1:10,000 instead, how many inches on the map would represent one mile in reality?

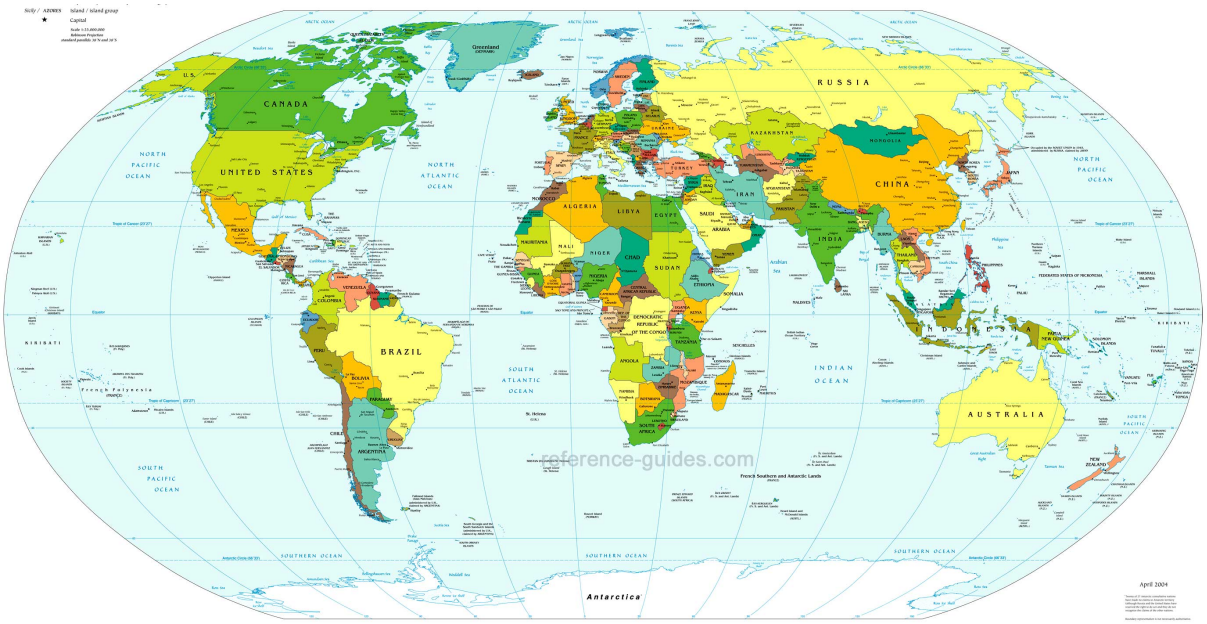
3. If your map instead had a scale factor of 1:100,000,000, how large would one mile be on it? Give your answer both in inches and in cm.

4. 1:100,000,000 is a common scale for world maps, indicating that the continents, oceans and countries that they show are **one hundred million** times smaller than their true size. The true circumference of the earth is about 25,000 miles. This means that one mile on the scale of world map is what fraction of the total width of the map? Represent this as a fraction, a decimal, and as a percentage.
   *Note: ALL world maps are approximately one Earth circumference wide so it does not matter what the scale factor of the map is. This **fraction** will always be the same!*

5. Is it practical to measure this distance on a typically sized (one large sheet of paper) world map? Why or why not?

6. What is a more realistic distance to show on a world map so that users can gauge the distance between objects on it? Describe how you arrived at this distance.

Consider the World Map shown below.



7. Draw and label the circumference of the earth on this map

8.
   a. Why is the map not a square?

   b. Think of one way in which this map might be misleading and describe it.

9. Using the map and the true value of the Earth's circumference, calculate it's scale factor. Show your work.